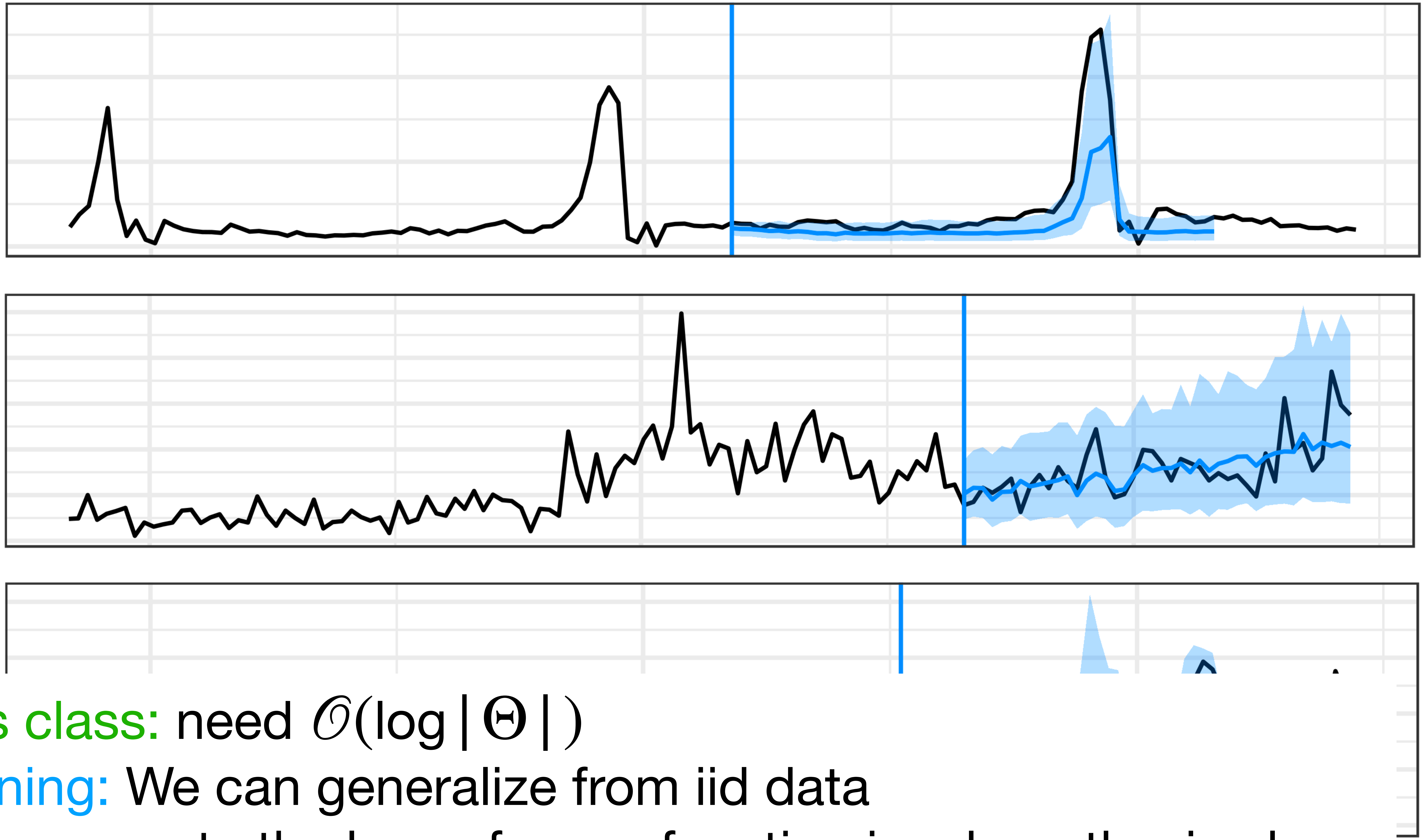
Reinforcement Learning for Supply Chains

Dhruv Madeka*, Dean Foster*, Sham M. Kakade^*

*Amazon ^Harvard University

First, let's look at supervised learning at Amazon.

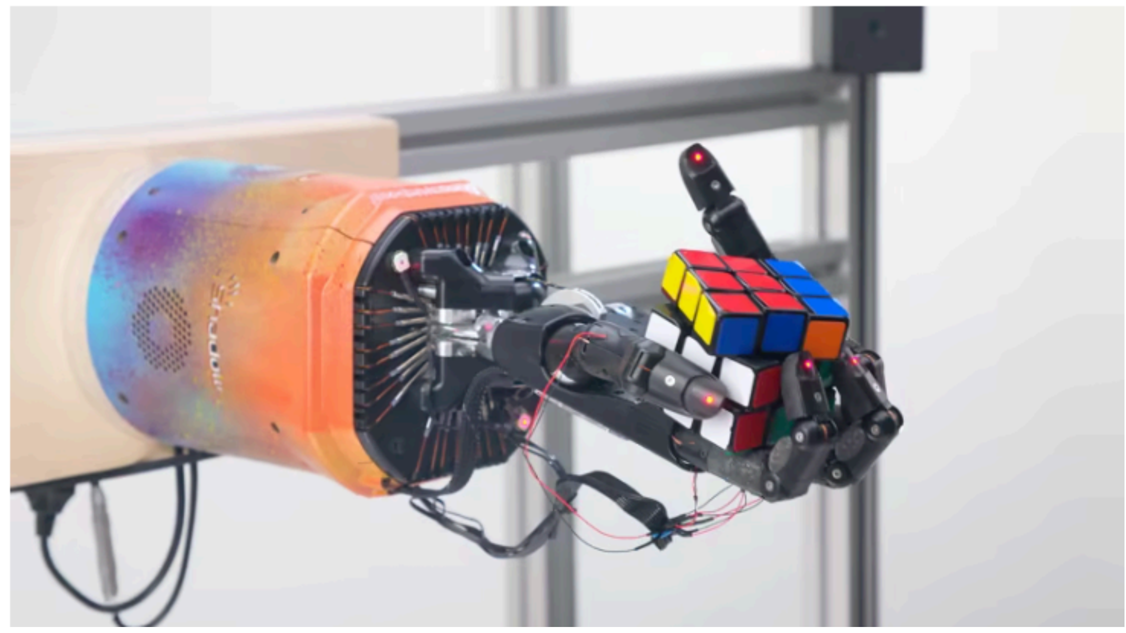


See Wen, Torkkola, Narayanaswamy, M. (2017)
Eisenach, Patel, M. (2020)

- Finite hypothesis class: need $\mathcal{O}(\log|\Theta|)$
- Supervised Learning: We can generalize from iid data

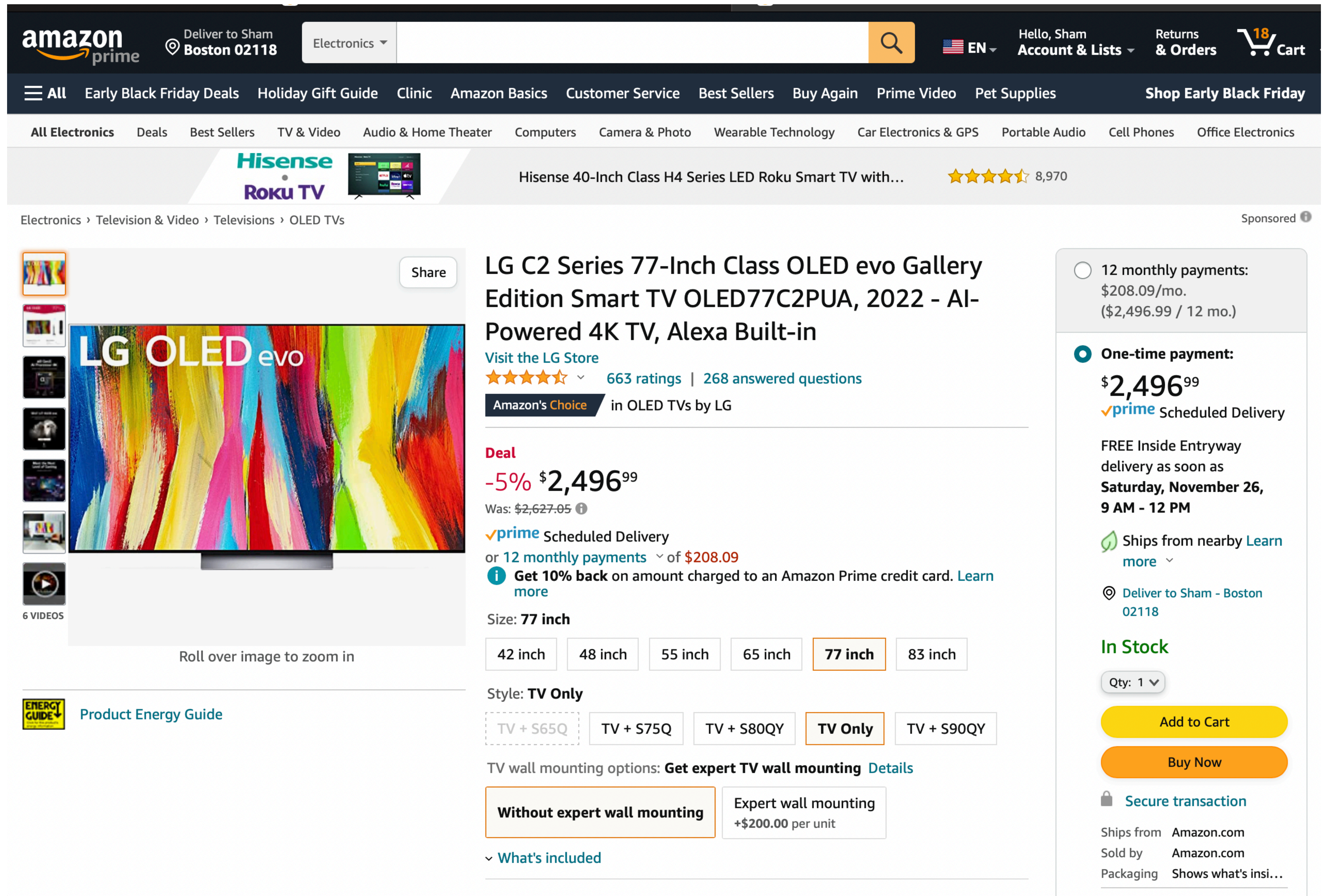
Data reuse: We can compute the loss of every function in a hypothesis class

Real-world RL is hard.



The core challenges Amazon faces are sequential decision making problems.

Can RL help in this space?



RL is hard!

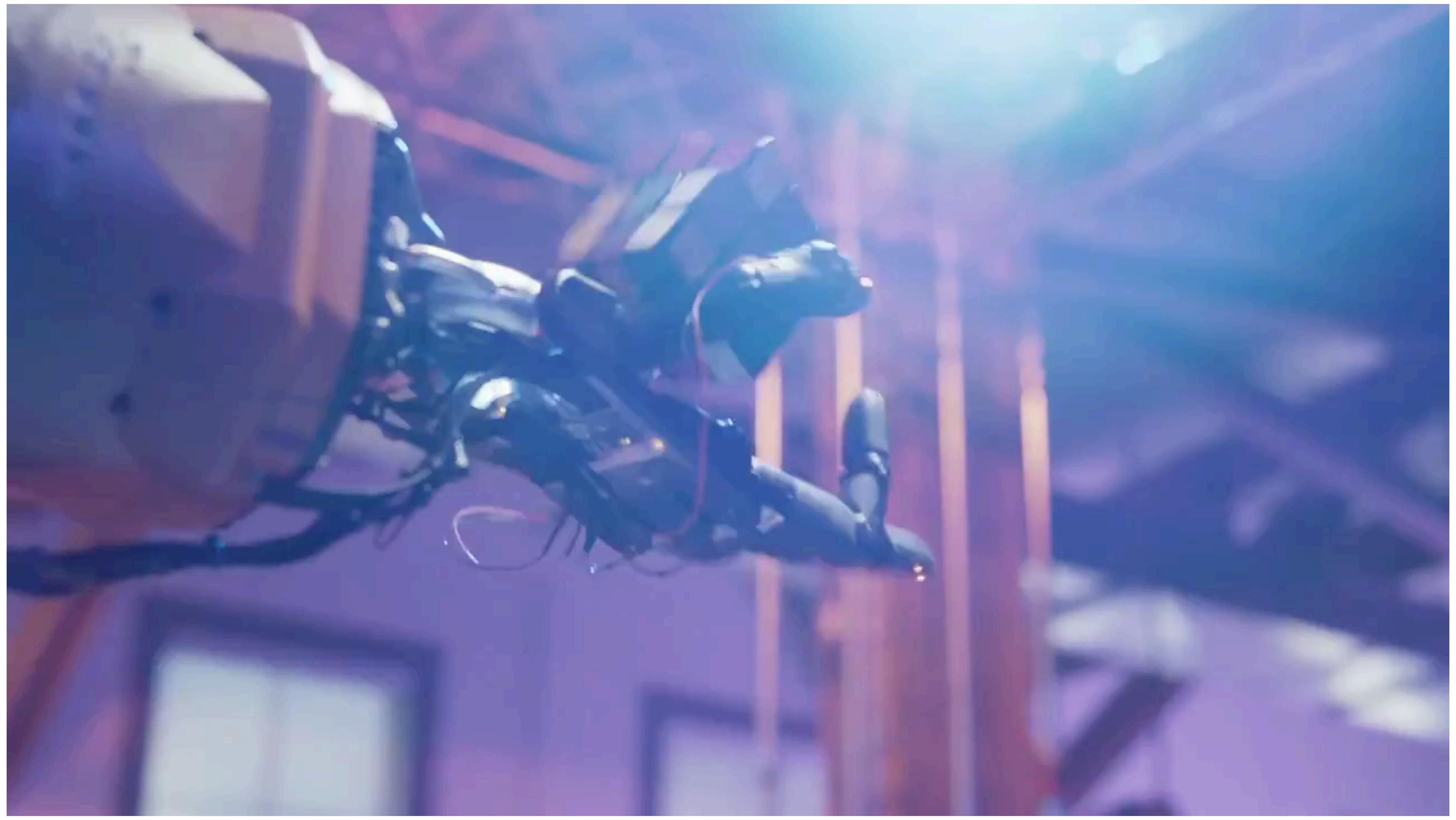
- Sample complexity can be as large as $\min(|\Theta|, 2^T)$

- Large state/action spaces

- Exploration

- Credit assignment problem

Dexterous Robotic Hand Manipulation
OpenAI, ‘19



The Supply Chain Problem

- Supply Chain is about buying, storing, and transporting goods.
- Amazon has been running it's Supply Chain for decades now
    - There is a lot of historical "off-policy" data
    - How do we use it?
    - Concern: counterfactual issue?
- This talk: how can we use this data to solve the inventory management problem?

Outline

Can we use historical data to solve inventory management problems in supply chain?

- Part I: Utilizing Historical Data
- Part II: Moving to real-world inventory management problems
- Part III: Real World Results

Deep Inventory Management

Dhruv Madeka
Amazon, maded@amazon.com
Kari Torkkola
Amazon, karito@amazon.com
Carson Eisenach
Amazon, ceisen@amazon.com
Anna Luo
Pinterest*, annaluo676@gmail.com
Dean P. Foster
Amazon, foster@amazon.com
Sham M. Kakade
Amazon, Harvard University, shamisme@amazon.com

Largely based on this paper: arxiv/2210.03137

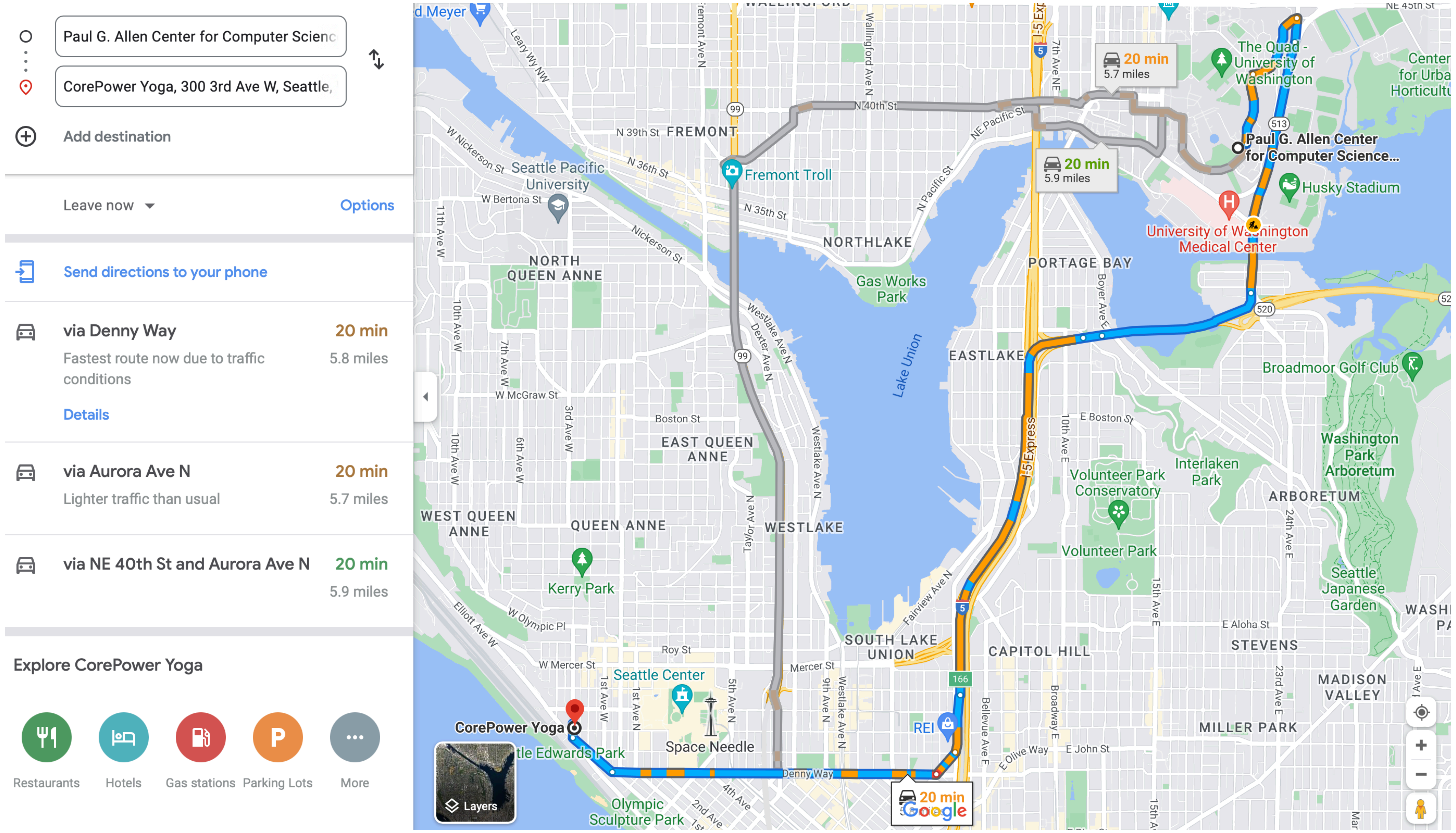
I: Utilizing historical data

Warm up: Vehicle Routing

(when using historical data might be ok)

- We want a good policy for routing a single car.

- Policy π: features -> directions
  features:
  time of day, holiday indicators, current traffic, sports games, accidents, location, weather,



- Historical Data:
  suppose we have logged historical data of features

- Backtesting policies:
  - Key idea: a single route minimally affects traffic
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

Warm up 2: Fleet Routing

- We want to route a whole fleet of self-driving taxis.

- Policy π: features -> directions
  - features:
    customer demand, time of day,
    holiday indicators, current traffic, sports games, accidents, location, weather…

- Historical Data:
  suppose we have logged historical data of features

- Backtesting policies:
  - Key idea: a small fleet route may have small affects on traffic.
  - Counterfactual: with the historical data, we can see what would have happened with another policy.

Supply Chain Data

Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	10	-10
1	90	20	-	40
1	70	-	50	-50
2	120	60	-	120
2	60	-	10	-10

Price= $2
Cost= $1

Backtesting a policy

Time	Inventory	Demand	Order	Revenue
0	100	20	-	40
0	80	-	~~10~~ 40	~~-10~~ -40
1	~~90~~ 120	20	-	40
1	~~70~~ 100	-	~~50~~ 20	~~-50~~ -20
2	120	60	-	120
2	60	-	10	-10

Price= $2
Cost= $1

- Current order doesn't impact future demand.
  - This allows us to backtest!
  - Empirically, backlog due to unmet demand does not look significant.[1]

1. See Verhoef et al (2006)

Formalization of the Supply Chain Problem

- Growing literature around a class of MDPs where a large part of the state is driven by an exogenous noise process [Efroni et al 2021, Sinclair et al 2022]

- A formalization of the model:
  - Action a_t: how much you buy
  - Exogenous random variables: evolving under $\Pr$ and not dependent on our actions $(\text{Demand}_t, \text{Price}_t, \text{Cost}_t, \text{Lead Time}_t, \text{Covariates}_t) := s_t$
  - Controllable part (inventory) I_t: evolution is dependent on our action.
    - $I_t = \min(I_{t-1} + a_{t-1} - D_t, 0)$ (and suppose we start at I_0).
  - Reward is just the sum of profits: $r(s_t, I_t, a_t) := \text{Price}_t \times \min(\text{Demand}_t, I_t) - \text{Cost}_t \times a_t$

- Learning setting:
  - Data collection: We observe N historical trajectories, where each sequence is sampled $s_1, \ldots, s_T \sim \Pr$
  - Goal: maximize our rewards cumulative reward over T periods

$$V_T(\pi) = E_\pi\left[\sum_{t=1}^{T} r(s_t, I_t, a_t)\right]$$

Why is it an interesting RL problem?

- Lots of time dependence!
    - If you buy too much, you’re left with the inventory for months!
    - Your actions (orders) affect the state at a random time later
    - Tons of correlation across time (Demand, Price, Cost)
- We can explore (linear risk in every product)

Theorem: Backtesting in ExoMDPs

Theorem [M., Torkkola, Eisenach, Luo, Foster, Kakade '22]:
Suppose we have a set of K policies $\Pi = \{\pi_1, \ldots \pi_K\}$, and we have N sampled exogenous paths. Then we can accurately backtest up to nearly $K \approx 2^N$ policies.

Formally, for any $\delta \in (0,1)$, with probability greater than $1 - \delta$ - we have that for all $\pi \in \Pi$:

$$|V_T(\pi) - \hat{V}_T(\pi)| \leq T\sqrt{\frac{\log(K/\delta)}{N}}$$

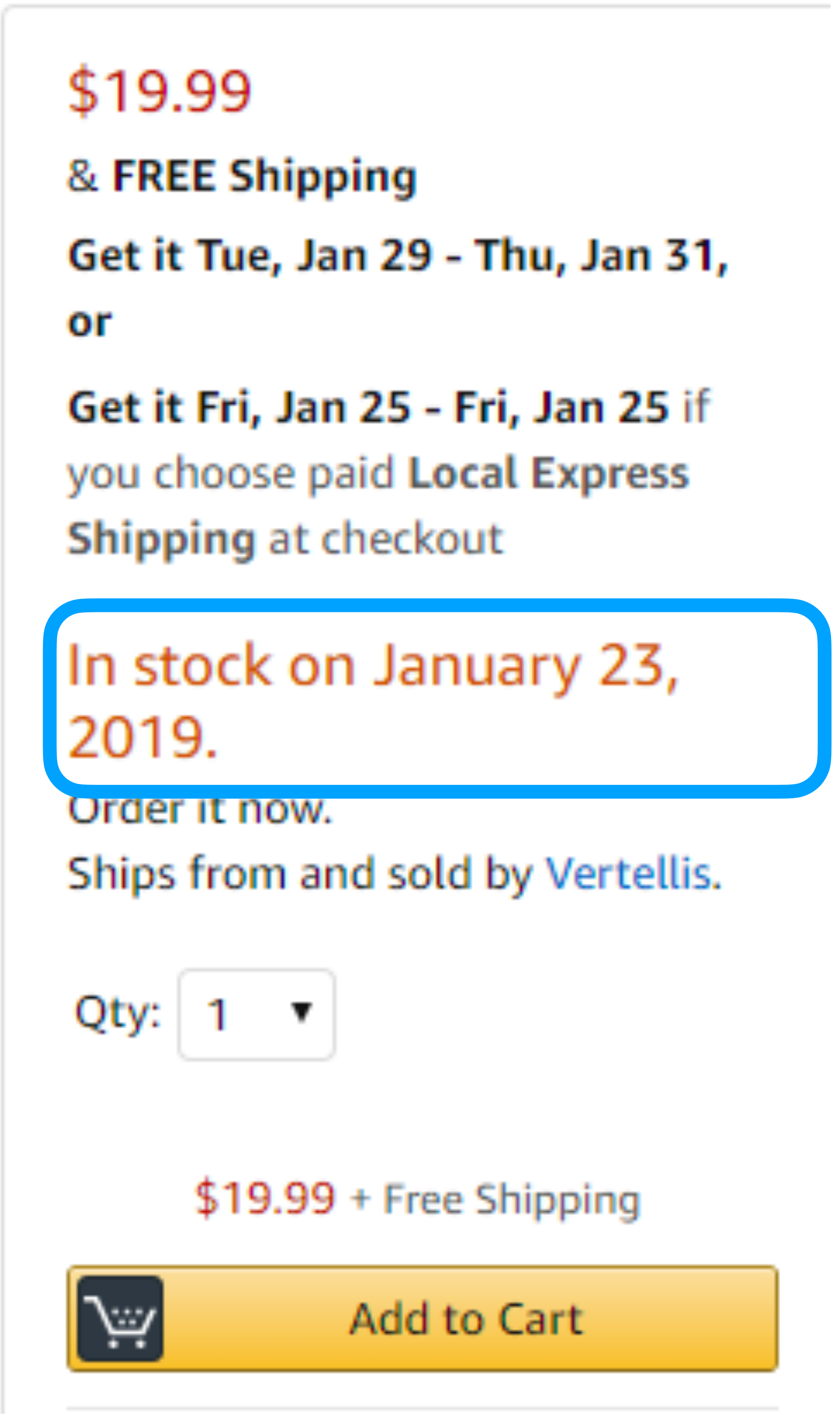
(assuming the reward r_t is bounded by 1).

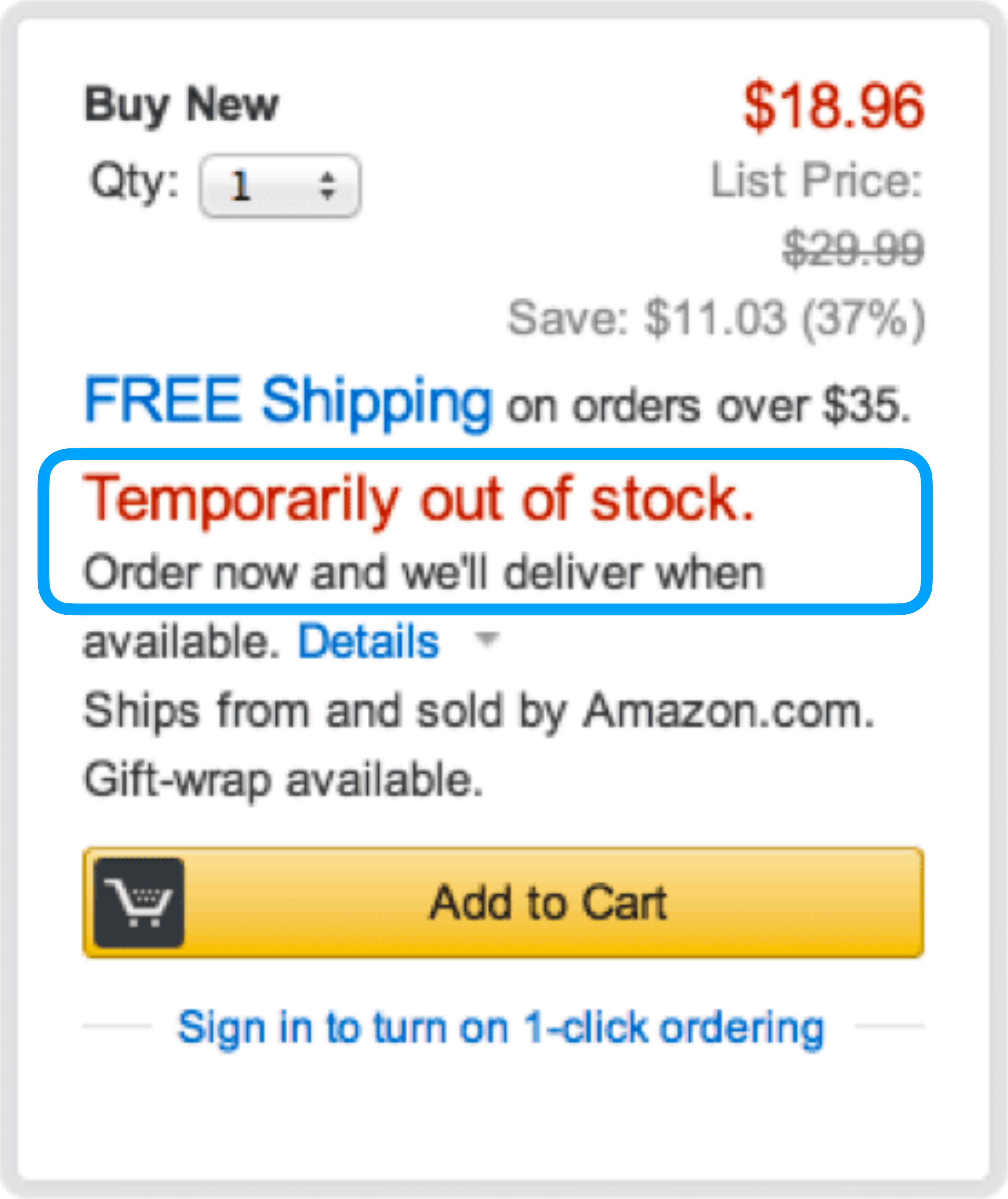
- Implications:
  - We can optimize a neural policy on the past data.
  - In the usual RL setting (not exogenous), we would have an amplification factor of (at least) $\min\{2^T, K\}$, using historical data due to the counterfactual issue.

II: Real World Inventory Management Problems

Real-world Issue: Censored Demand

- When demand $\geq$ inventory, what customers see:





We only observe **sales** not the **demand:**

Sales $:=$ min(Demand, Inventory)

Can we still backtest?

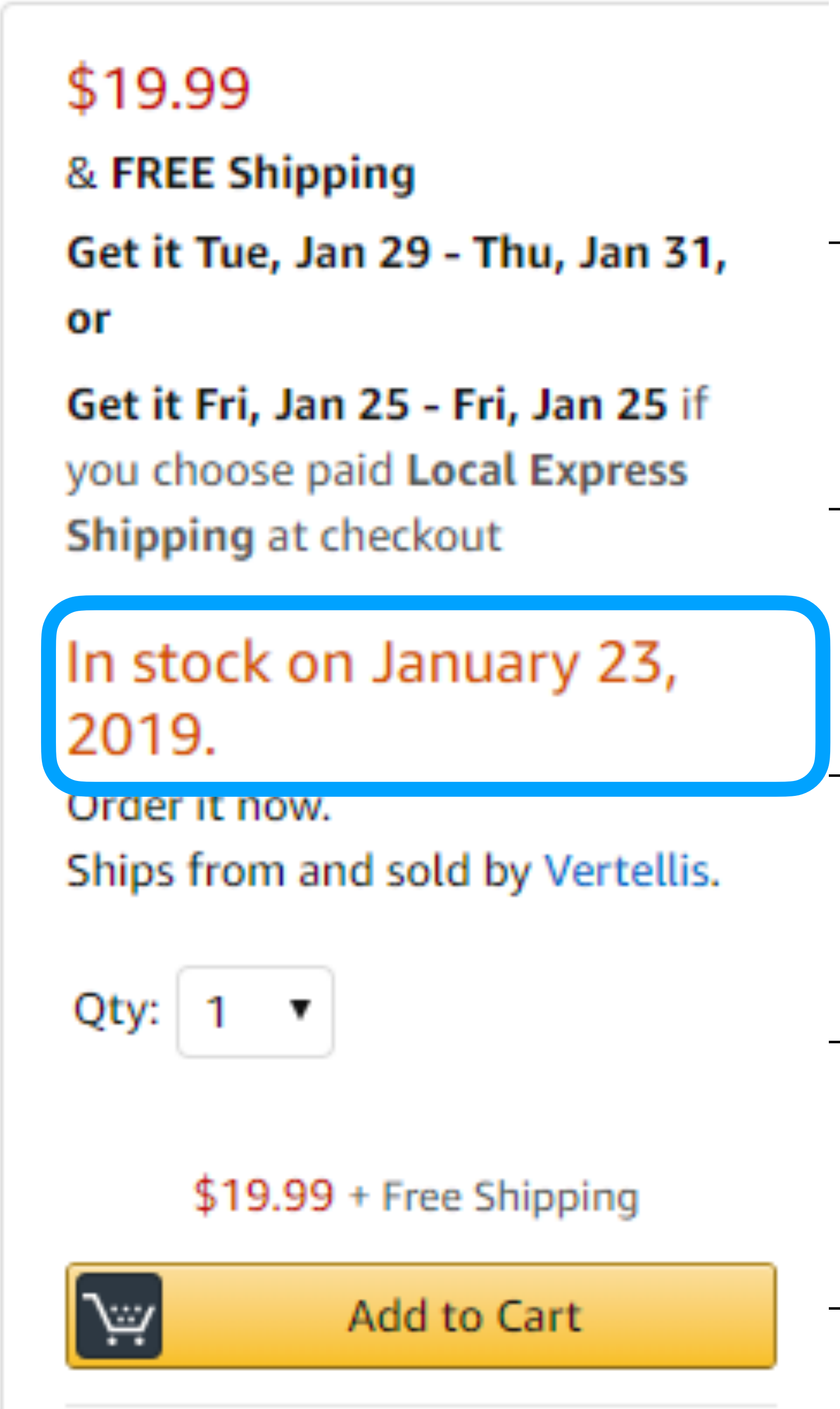
Our historical data is then censored….

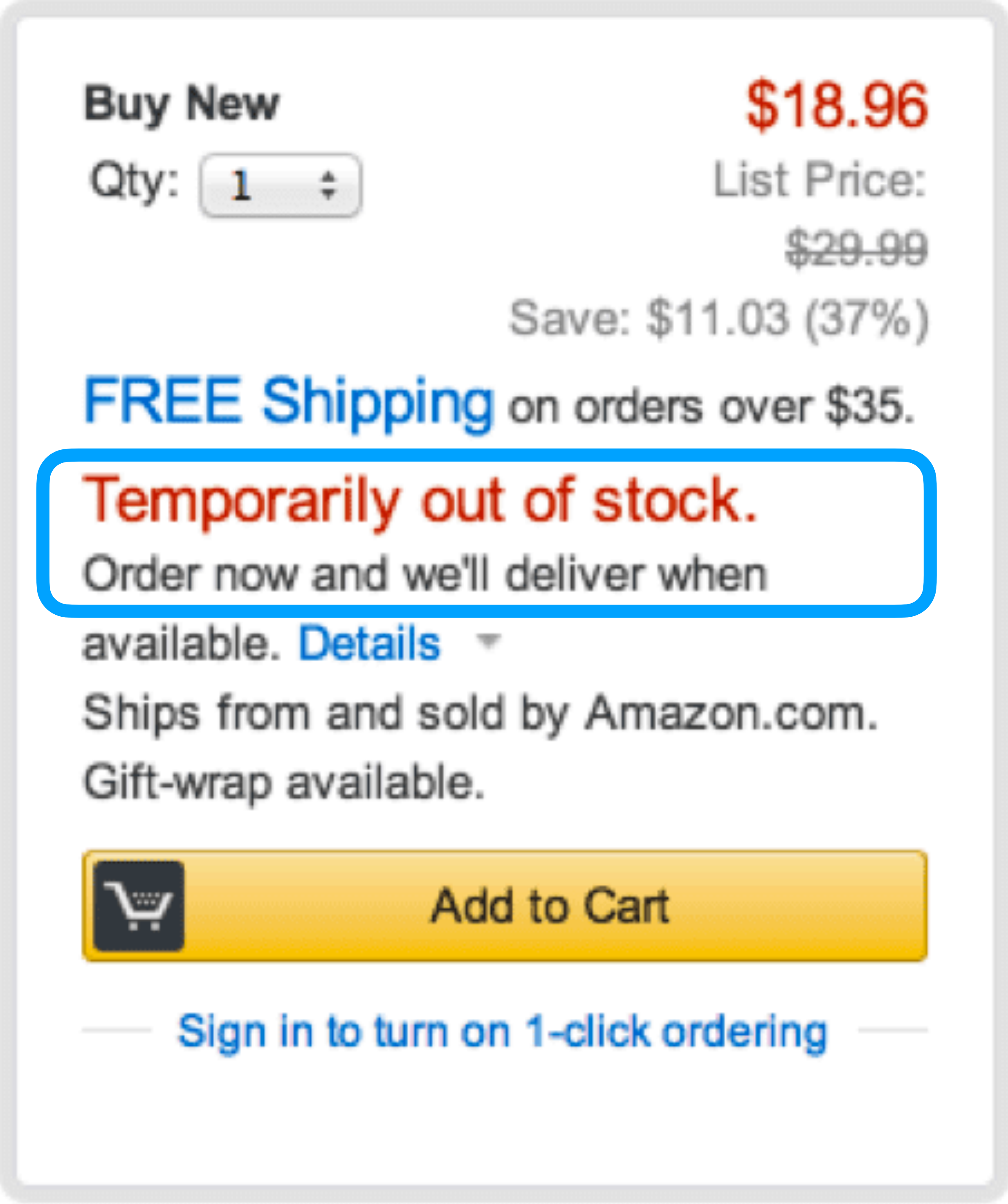
$$\text{Sales} := \min(\text{Demand}, \text{Inventory})$$

Price= $2

Cost= $1

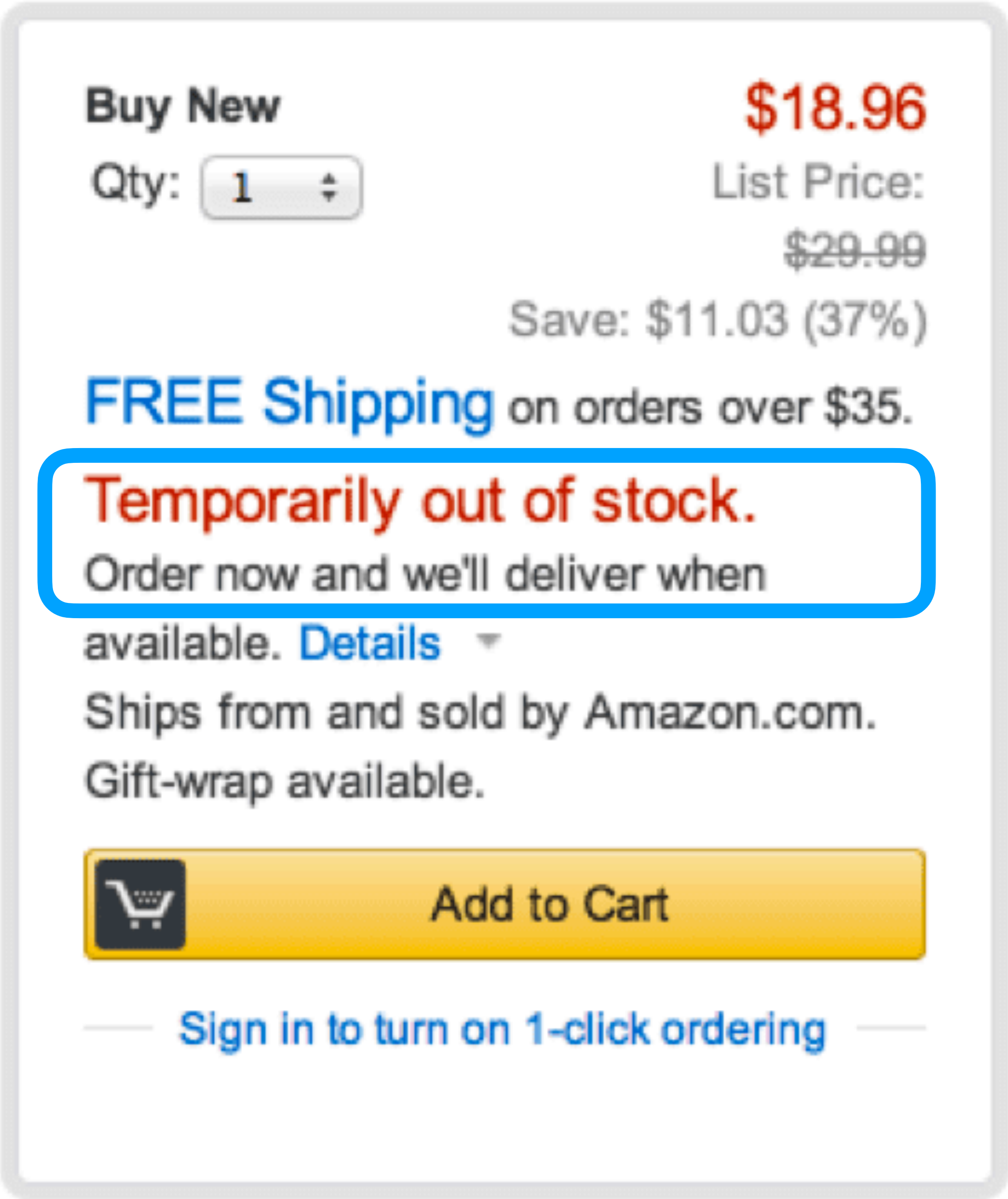
Time	Inventory	True Demand	Sales	Order	Revenue
T	10	??	10	-	20
⋮	⋮				
⋮	⋮				
⋮	⋮				
⋮	⋮				
⋮	⋮				





If we could fill in the missing demand, then we could still backtest!

We have many observed historical covariates



- Covariates:
  Sales, Web Site, Glance Views, Product Text, Reviews

- Example: the #times customers look at an item gives us info about the unobserved demand.

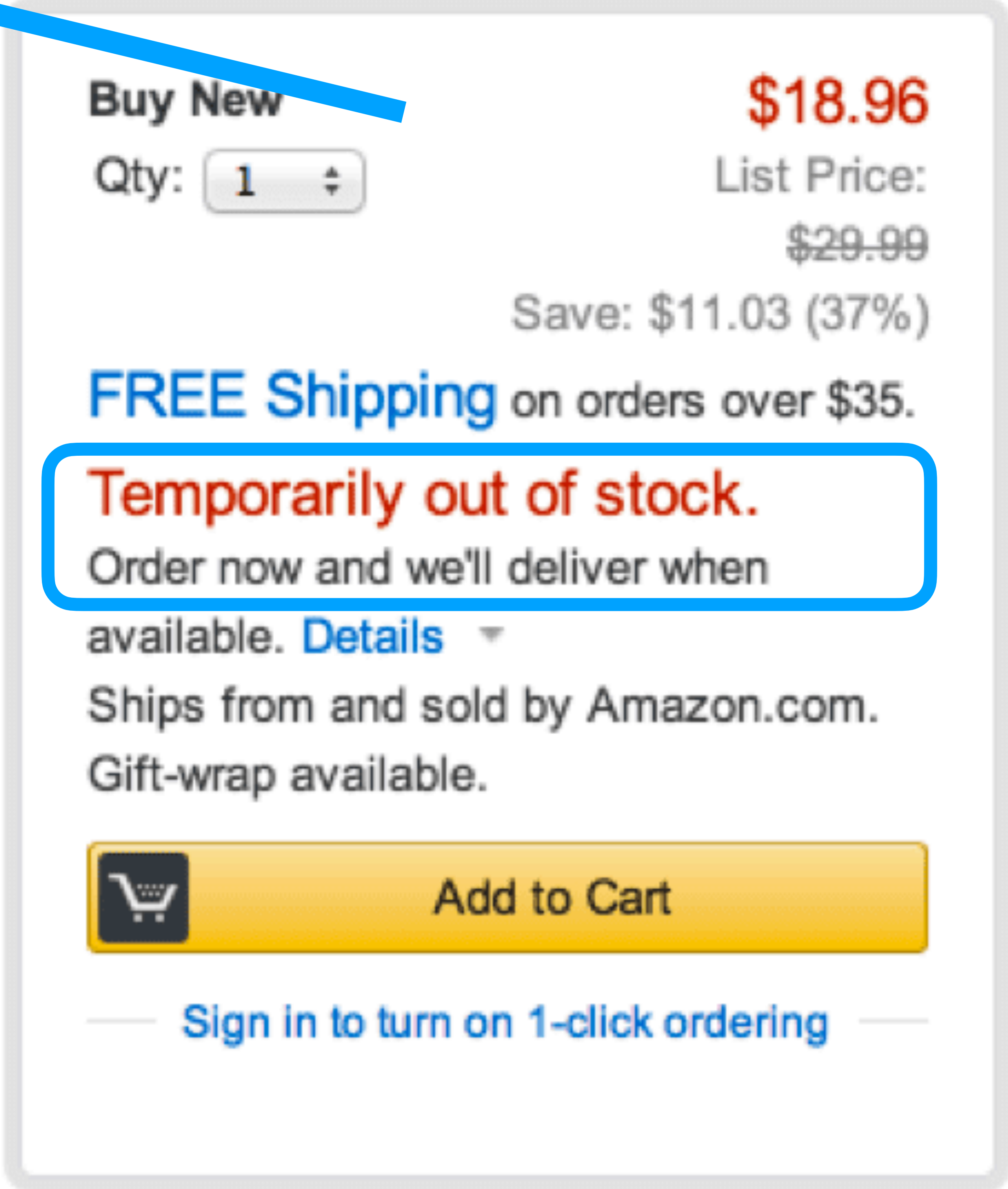
- Let's forecast the missing variables from the observed covariates!
  $\hat{\mathbb{P}}(\text{Missing Data} \mid \text{Observed Data})$

Uncensoring the data….

Sales := min(Demand, Inventory)

Price= $2
Cost= $1

Time	Inventory	True Demand	Sales	Order	Revenue
T	10	**40**	10	-	20
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		
⋮	⋮	⋮	⋮		



Key idea: Use covariates (e.g. glance views) to forecast missing demand, vendor lead times, etc

Theorem: Backtesting in Uncensored ExoMDPs

Theorem [M., Torkkola, Eisenach, Luo, Foster, Kakade 22]:
If we can forecast the missing variables accurately (in a total variation sense), then we can backtest accurately. More formally,

Setting: we have N sampled sequences $\{s_1^i, s_2^i, \ldots s_T^i\}_{i=1}^N$,
where M_i and O_i are the missing and observed exogenous variables in sequence i.

Forecast: $\widehat{\mathbb{P}}^i = \widehat{\Pr}(M_i \mid O_i)$ is our forecast of $\mathbb{P}^i = \Pr(M_i \mid O_i)$.

Assume: With pr. 1, forecasting has low error: $\frac{1}{N}\sum_{i=1}^{N} \mathrm{TotalVar}\left(\mathbb{P}^i, \widehat{\mathbb{P}}^i\right) \leq \epsilon_{\mathrm{sup}}$.

Guarantee: For any $\delta \in (0,1)$, with pr. greater than $1-\delta$, for all $\pi \in \Pi$:

$$|V_T(\pi) - \hat{V}_T(\pi)| \leq T\left(\epsilon_{\mathrm{sup}} + \sqrt{\frac{\log(K/\delta)}{N}}\right)$$

- Key idea: We can backtest even in the censored setting!

III: Training Policies & Empirical Results

The Simulator

- Collection of historical trajectories:
  - 1 million products
  - 104 weeks of data per product

Data

- Uncensoring:
  - Demand
  - Vendor Lead Times

Corrections

- Policy gradient methods in a “gym”:
  - “gym” ↔ backtesting ↔ simulator (note the “simulator” isn’t a good world model).
  - The policy can depend on many features. (seasonality, holiday indicators, demand history, ASIN, text features)

Simulator

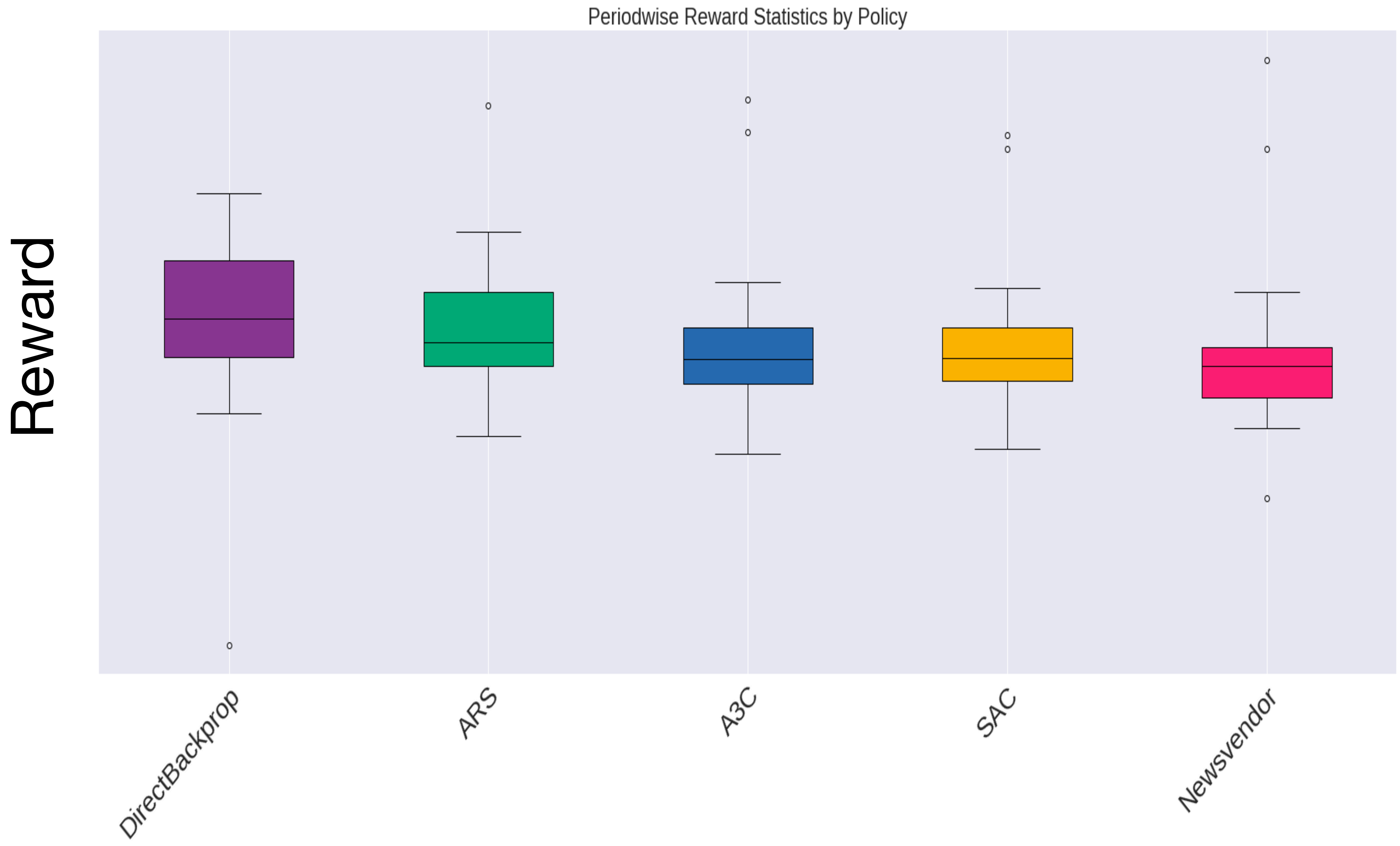
Differentiable Control Problem

- Note that each term of our state evolution is a differentiable function of previous actions
- So, we can take gradients directly from our Reward through our policy
- This is our current production policy, called *DirectBackprop*
- Similar in spirit to Perturbation Analysis (Glasserman et al 1995), except it uses a neural policy

Sim to Real Transfer

- Sim: the backtest of DirectBackprop improves on Newsvendor.
- Real: DirectBackprop significantly reduces inventory without significantly reducing total revenue.

Simulation



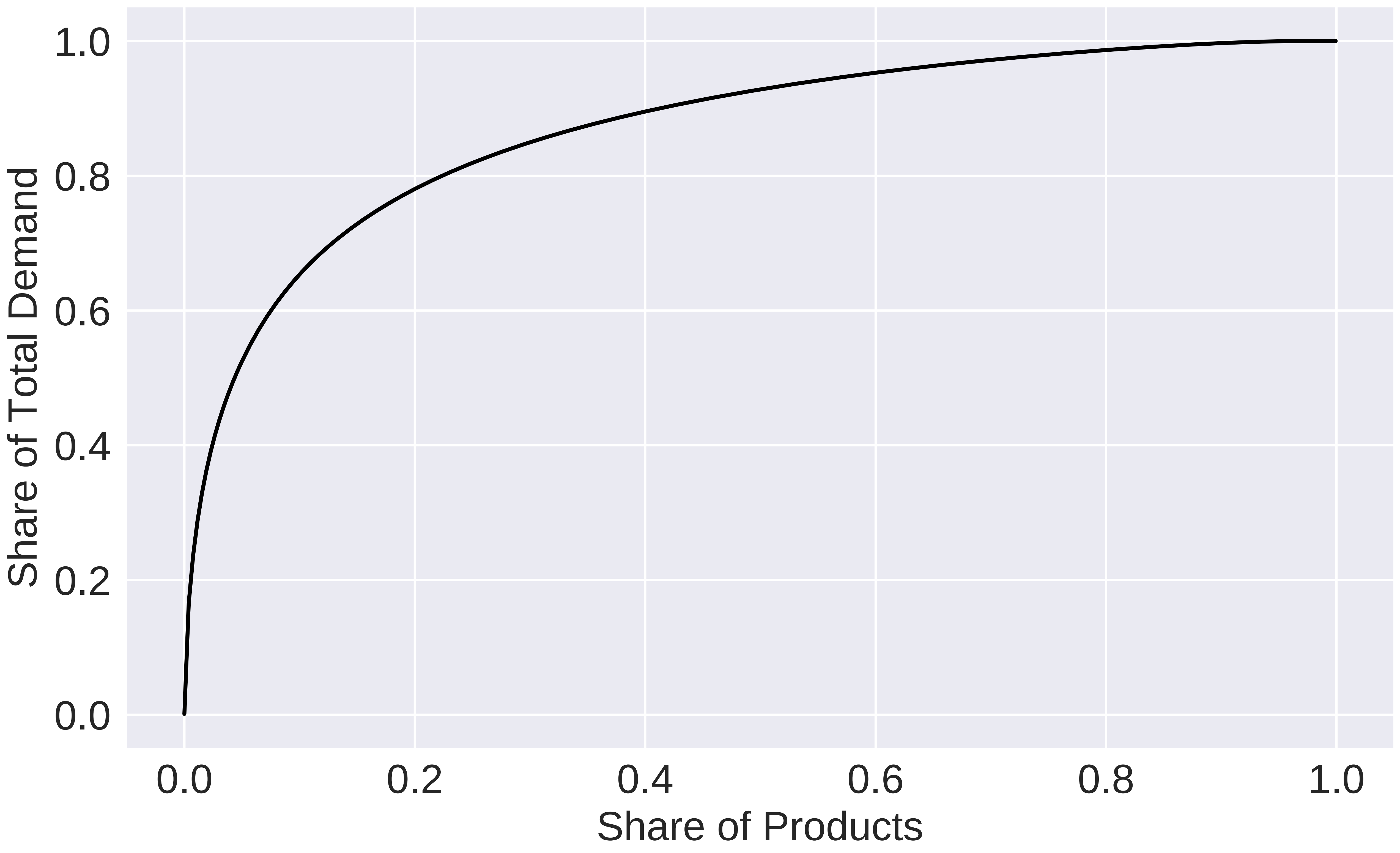
Real World

Metrics	% change
Inventory Level	**-12** $\pm$ 6
Revenue	$\sim$

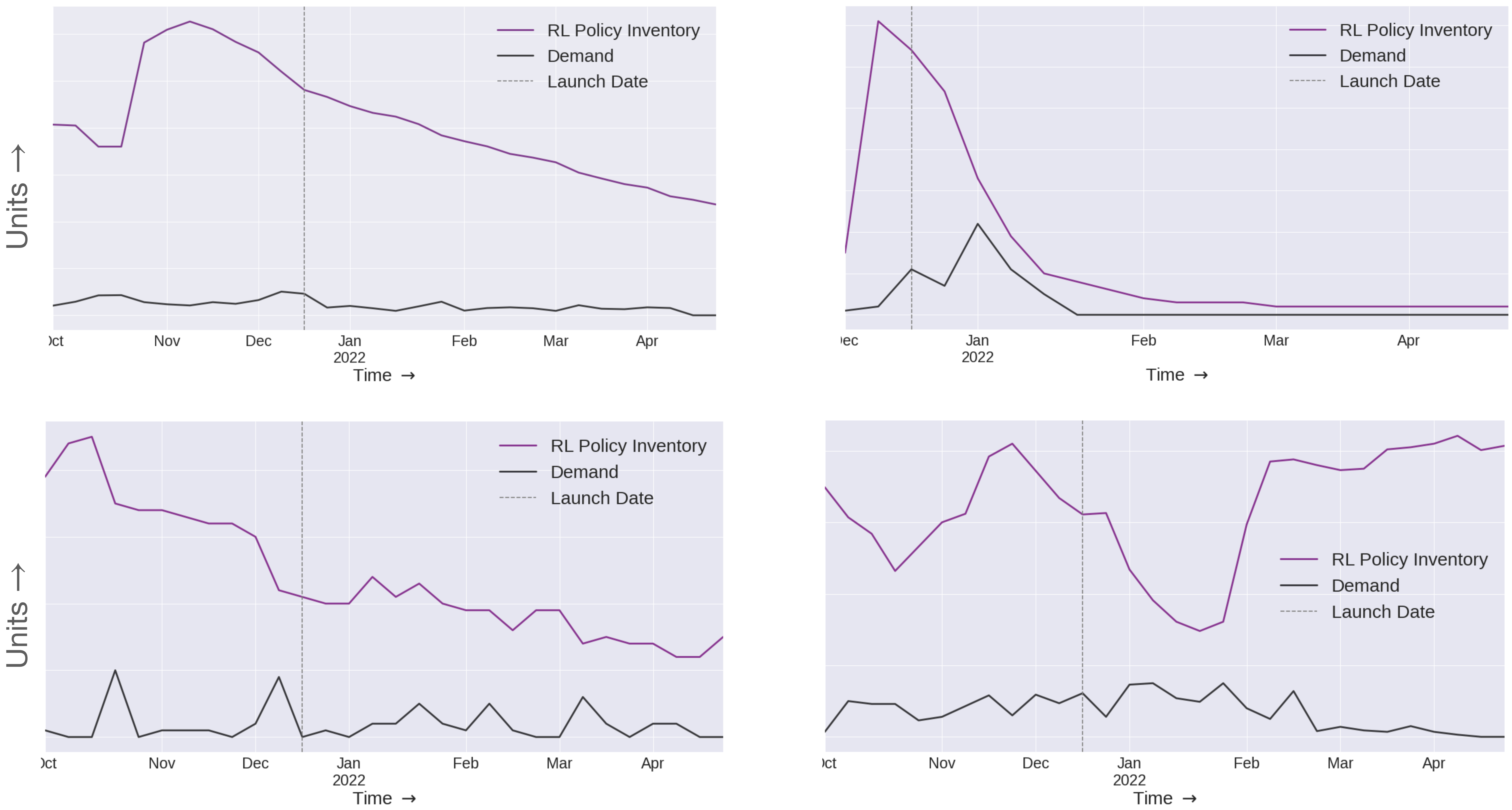
What about in the real world?

- Really hard to measure! (Tripuraneni, M et. al 2021)

- Heavy tailed data:
  - A few products contribute to most of the reward



Anecdotally, RL has reasonable strategies in the real world…



Real World RL Challenges

- World is not perfectly exogenous (some terms may depend on our actions)
- Cross product constraints are computationally intensive
- Not every Supply Chain problem can be written in this framework

Conclusion

- There are a class of RL Problems that work in the real world!

- The exogenous assumption allows us to backtest **any** policy on historical data

- A large number of classical Operations Research problems fall into this class of Interactive Decision-Making problems

Carson

Kari

Anna

Dean

Sham